Online Supplementary Appendix to

"Should Buyers or Sellers Organize Trade in a Frictional Market?"

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Abstract: This online appendix provides extends the baseline model and provides additional proofs of the results in the paper. The sections are numbered sequentially from the appendix to the paper. Section D proves Theorem C.1 that states the results when the regularity condition (Assumption 3) is violated. Section E allows for unequal welfare weights on the two sides of the market and proves that most of the results in the baseline model continue to hold. Section F discusses how trading platforms can affect the results. Section G introduces search effort on the inelastic side to make the supply partially elastic and proves that all main results of the paper remain valid.

D. Proof of Theorem C.1

Let $\hat{c}$ be any generic interior solution of c to $G(c,k)=0$. That is, $G(\hat{c},k)=0$, $G'_c(\hat{c},k)\neq 0$, and $\hat{c}\in(0,\bar{c})$. Define $\Psi(c)=f(c+k)$. Then, $\Psi(\hat{c})$ is also a generic interior solution of c to $G(c,k)=0$. To prove this, use the definition of $\hat{c}$ to obtain $f(\hat{c}+k)+k=f^{-1}(\hat{c})$. Then, $f(\Psi(\hat{c})+k)=f(f^{-1}(\hat{c}))=\hat{c}$, i.e., $\Psi(\Psi(\hat{c}))=\hat{c}$. We have:

$$G(\Psi(\hat{c}),k)=f^{-1}(\Psi(\hat{c}))-k-\hat{c}=0.$$

Since $\hat{c} > 0$, then $\Psi(\hat{c}) = f(\hat{c} + k) < f(k) \leq \bar{c}$, where the property $f' < 0$ is used. If $f(k) \leq A - k$, then $\hat{c} < \bar{c}$ implies $\hat{c} < A - k$ and, hence, $\Psi(\hat{c}) = f(\hat{c} + k) > f(A) = 0$. If $f(k) > A - k$, then $\hat{c} < \bar{c}$ implies $\hat{c} < f(k)$ and, hence, $\Psi(\hat{c}) = f(\hat{c} + k) = f^{-1}(\hat{c}) - k > 0$. Thus, $\Psi(\hat{c})$ is interior. To prove that $\Psi(\hat{c})$ is generic, use the definition of $\hat{c}$ and Lemma A.1 to derive:

$$\frac{1}{\theta_n(\hat{c})} = h^{-1}(\hat{c}) = \theta_e\left(F'\left(h^{-1}(\hat{c})\right)\right) = \theta_e\left(f^{-1}(\hat{c})\right) = \theta_e(\Psi(\hat{c}) + k).$$

The first equality comes from $\theta_n(.)$ defined in (2.6), the second equality from $\theta_e(.)$ defined in (2.2), the third equality from (A.3), and the last equality from $G(\hat{c}, k) = 0$. Because $\hat{c} = \Psi(\Psi(\hat{c}))$, the above result implies $\frac{1}{\theta_n(\Psi(\hat{c}))} = \theta_e(\hat{c} + k)$. Recall that $G'_c(c, k) = \theta_e(c + k) - \theta_n(c)$ for all c (see (iii) in Lemma A.2). Since $G'_c(\hat{c}, k) \neq 0$, then

$$\begin{aligned} &G'_c(\Psi(\hat{c}), k) = \theta_e(\Psi(\hat{c}) + k) - \theta_n(\Psi(\hat{c})) \\ &= \tfrac{1}{\theta_n(\hat{c})} - \tfrac{1}{\theta_e(\hat{c}+k)} = \tfrac{G'_c(\hat{c},k)}{\theta_n(\hat{c})\theta_e(\hat{c}+k)} \neq 0. \end{aligned}$$

Therefore, $\Psi(\hat{c})$ is a generic solution of c to $G(c, k) = 0$.

The mapping Ψ has a unique fixed point, which is $c_d \in (0, \bar{c})$. Since $G'_c(c_d, k)$ has the same sign as $(k_0 - k)$ (see (i) in Lemma A.3), the assumption $k \neq k_0$ implies that c_d is a generic interior solution. For any solution $\hat{c}$, if $\hat{c} < c_d$, then $\Psi(\hat{c}) > f(c_d + k) = c_d$; if $\hat{c} > c_d$, then $\Psi(\hat{c}) < c_d$. Since every solution on one side of c_d has an image on the other side of c_d that is also a solution, the number of solutions of c to $G(c, k) = 0$ (including c_d) is odd. This result also holds for the number of generic interior solutions. Let $J - 1 \geq 0$ be the number of generic interior solutions on each side of c_d. The set of generic interior solutions is $\{c_{J+j}\}_{j=-(J-1)}^{J-1}$ with $c_1 < c_2 < ... < c_{2J-1}$, where $c_J = c_d$. Then, $\Psi(c_{J+j}) = c_{J-j}$ for all $j = -(J-1), ..., (J-1)$.

As in the proof of Theorem 3.1, if $k = 0$ and the matching function is symmetric, then social welfare is the same under the two organizations. Note that the current theorem does not need the condition on $(k - k_0)[A - k - f(k)]$ as in the proof of Theorem 4.1. This condition was used there for consistency with Assumption 3 which is not imposed here.

(i) The case $k > k_0$: In this case, $G'_c(c_d, k) < 0 = G(c_d, k)$. Since $G(c_d - \varepsilon, k) > 0$ for sufficiently small $\varepsilon > 0$, then $G(c, k) > 0$ for $c \in (c_{J-1}, c_J)$. If $J \geq 2$, then c_{J-1} is a generic interior solution, and $G'_c(c_{J-1}, k) > 0$. This further implies $G(c, k) < 0$ for $c \in (c_{J-2}, c_{J-1})$. Repeating this argument, we can prove that $G(c, k) > 0$ for $c \in (c_{J+2\ell-1}, c_{J+2\ell})$ where $\ell = -\left\lfloor \frac{J-1}{2} \right\rfloor, ..., 0$, and that $G(c, k) < 0$ for $c \in (c_{J+2\ell}, c_{J+2\ell+1})$, where $\ell = -\left\lfloor \frac{J}{2} \right\rfloor, ..., -1$. Applying a similar argument on the right side of c_d yields that $G(c, k) < 0$ for $c \in (c_{J+2\ell}, c_{J+2\ell+1})$ where $\ell = 0, ..., \left\lfloor \frac{J}{2} \right\rfloor - 1$, and that $G(c, k) > 0$ for $c \in (c_{J+2\ell-1}, c_{J+2\ell})$ where $\ell = 1, ..., \left\lfloor \frac{J-1}{2} \right\rfloor + 1$. Putting the results on the two sides of c_d together, we have $G(c, k) > 0$ for $c \in \Omega_1$ and $G(c, k) < 0$ for $c \in \Omega_2$, where Ω_1 and Ω_2 are defined in (C.1). That is, the efficient organizers are inelastic if $c_e \in \Omega_1$ and elastic if $c_e \in \Omega_2$, as stated in (i).

(ii) The case $k < k_0$: In this case, $G'_c(c_d, k) > 0 = G(c_d, k)$. Then, $G(c, k) > 0$ for $c \in \Omega_2$ and $G(c, k) < 0$ for $c \in \Omega_1$. Consequently, the efficient organizers are elastic if $c_e \in \Omega_1$ and inelastic if $c_e \in \Omega_2$, as stated described in (ii).

To analyze which side of the market is short, recall $G'_c(c, k) = \theta_e(c + k) - \theta_n(c)$ for all c (see (iii) in Lemma A.2). Consider the case with $k > k_0$, i.e., case (i) of the current theorem. Since $k > k_0$, then $G'_c(c_d, k) < 0$ and $\theta_e(c_d + k) = \frac{1}{\theta_n(c_d)} < 1$, as shown in the proof of

Theorem 3.1. Consider any c_e in the interval (c_{J-1}, c_J), where $c_J = c_d$. In this interval, $G(c_e, k) > 0$ and trade should be organized by inelastic individuals. Because $c_e < c_d$ in this interval and $\theta_n(c)$ is a decreasing function, the site-visitor ratio is $\frac{1}{\theta_n(c_e)} < \frac{1}{\theta_n(c_d)} < 1$. That is, the efficient market organizers are on the short side. Note that $G(c_e, k)(c_e - c_d) < 0$ for $c_e \in (c_{J-1}, c_J)$. Now consider any c_e in the interval (c_{J-2}, c_{J-1}). In this interval, $G(c_e, k) < 0$ and trade should be organized by elastic individuals. Also, $G'_c(c_{J-1}, k) > 0$, because G'_c must change signs when c changes from c_J to c_{J-1}. Since $\theta_n(c_{J-1}) > 1$, as shown above, then $G'_c(c_{J-1}, k) > 0$ implies $\theta_e(c_{J-1} + k) > \theta_n(c_{J-1}) > 1$. Thus, for $c_e \in (c_{J-2}, c_{J-1})$, the site-visitor ratio is $\theta_e(c_e + k) > \theta_e(c_{J-1} + k) > 1$. That is, the efficient market organizers are on the long side. Note that $G(c_e, k)(c_e - c_d) > 0$ in this interval. By induction, we can prove the following result for all $c_e < c_d$: the market organizers should be on the short side if and only if $G(c_e, k)(c_e - c_d) < 0$, and on the long side if and only if $G(c_e, k)(c_e - c_d) > 0$. A similar proof shows that the result also holds for $c_e > c_d$. Moreover, the result holds in the case $k < k_0$, i.e., case (ii) of the current theorem. **QED**

E. Proof of Proposition 6.2

The welfare weights are $\lambda \neq 1$ for elastic individuals relative to inelastic individuals. Let σ be the surplus share chosen by the planner for the elastic side in a match, which matters for social welfare when $\lambda \neq 1$. In market e, the expected surplus of participating in the market is $[\frac{F(\theta)}{\theta}\sigma - (c_e + k)]$ for an elastic individual and $[F(\theta)(1 - \sigma) - c_n]$ for an inelastic individual. Since the measure of elastic individual is θ and the measure of inelastic individuals is 1, social welfare measured as the weighted sum of expected surpluses over all individuals is:

$$w_e(\theta,\sigma) = \lambda\left[F(\theta)\sigma - (c_e+k)\theta\right] + \left[F(\theta)(1-\sigma) - c_n\right].$$

The planner chooses (θ,σ) to maximize $w_e(\theta,\sigma)$ subject to individual rationality constraints that the expected surplus of participating in the market should be non-negative. This constraint is $\sigma \geq \sigma_{Le}$ for side e and $\sigma \leq \sigma_{He}$ for side n, where

$$\sigma_{Le} \equiv \frac{\theta}{F(\theta)}(c_e+k),\ \sigma_{He} \equiv 1 - \frac{c_n}{F(\theta)}.$$

If market e is viable, then the interval $[\sigma_{Le},\sigma_{He}]$ is non-empty. Rewrite w_e as

$$w_e(\theta,\sigma) = [\lambda + (1-\lambda)(1-\sigma)]F(\theta) - \lambda(c_e+k)\theta - c_n.$$

It is clear that the socially efficient σ maximizes $(1-\lambda)(1-\sigma)$ under the constraints $\sigma_{Le} \leq \sigma \leq \sigma_{He}$. For any $\lambda \neq 1$, one of the two individual rationality constraints is binding. Precisely, the efficient σ is $\sigma = \sigma_{Le}$ if $\lambda < 1$, and $\sigma = \sigma_{He}$ if $\lambda > 1$. With this division of the match surplus, social welfare is

$$\hat{w}_e(\theta) = [F(\theta) - (c_e+k)\theta - c_n]\max\{\lambda, 1\}.$$

The expression in [.] is $w_e(\theta)$ under equal welfare weights (see (2.1)). Thus, the efficient θ in market e under any relative welfare weight λ is identical to that under $\lambda = 1$.

The analysis for market n is similar. In market n, the expected surplus of participating in the market is $\left[F(\frac{1}{\theta})\sigma - c_e\right]$ for an elastic individual and $\left[\theta F(\frac{1}{\theta})(1-\sigma) - k - c_n\right]$ for an inelastic individual. Social welfare is

$$w_n(\theta,\sigma) = \lambda\left[\theta F(\frac{1}{\theta})\sigma - c_e\theta\right] + \left[\theta F(\frac{1}{\theta})(1-\sigma) - k - c_n\right].$$

The planner chooses (θ, σ) to maximize $w_n(\theta, \sigma)$ subject to individual rationality constraint $\sigma \geq \sigma_{Ln}$ for side e and $\sigma \leq \sigma_{Hn}$ for side n, where

$$\sigma_{Ln} \equiv \frac{c_e}{F(\frac{1}{\theta})}, \ \sigma_{Hn} \equiv 1 - \frac{c_n + k}{\theta F(\frac{1}{\theta})}.$$

The efficient σ is $\sigma = \sigma_{Ln}$ if $\lambda < 1$, and $\sigma = \sigma_{Hn}$ if $\lambda > 1$. With this efficient division, social welfare is

$$\hat{w}_n(\theta) = \left[\theta F(\frac{1}{\theta}) - c_e\theta - c_n - k\right] \max\{\lambda, 1\}.$$

The expression in [.] is $w_n(\theta)$ under equal welfare weights (see (2.5)). Thus, the efficient θ in market n under any relative welfare weight λ is identical to that under $\lambda = 1$.

In the social optimum, maximized welfare is $W_e \max\{\lambda, 1\}$ in market e and $W_n \max\{\lambda, 1\}$ in market n, where W_e is given by (2.3) and W_n by (2.7). The market that yields higher welfare is the efficient organization. Since comparing welfare between the two markets is the same as comparing W_e and W_n, the efficient market organization under any $\lambda \in (0, \infty)$ is the same as in the benchmark model where $\lambda = 1$.

The equilibrium with directed search in section 6 maximizes the inelastic side's expected surplus of participating in the market subject to the constraint that the expected surplus is zero for the elastic side. As shown in section 6, the equilibrium θ_i maximizes $w_i(\theta)$, and the market organization that wins the competition is the one with a higher W. Thus, θ and the market organization in the equilibrium are the same as in the above social optimum. However, equilibrium welfare is the same as in the above social optimum only when $\lambda \leq 1$, in which case the social optimum yields zero expected surplus to the elastic side. When $\lambda > 1$, the social optimum requires the expected surplus to be positive for elastic individuals and zero for inelastic individuals, in contrast to the equilibrium where the expected surplus is zero for elastic individuals and positive for inelastic individuals. Also, in this case, overall welfare in the economy is equal to $\lambda \max\{W_e, W_n\}$ in the social optimum, which is higher than the equilibrium counterpart, $\max\{W_e, W_n\}$. **QED**

F. Trading Platforms

In this appendix, we incorporate trading platforms created by a third party. Besides network externalities, which we have abstracted from, a trading platform can differ from the trading sites in the benchmark model in subsection 2.1 in the following aspects:

(i) Neither side of the market incurs the site cost. Instead, the platform incurs the site cost and charges fees on the users. The fees add to the participation cost.

(ii) The capacity constraint on sites may be different.

(iii) The matching function and the site cost may be different. For example, a trading platform can change meetings from one-to-one to other forms or reduce the site cost.

We show that element (i) does not change the efficient allocation. Not surprisingly, elements (ii) and (iii) affect the efficient allocation, because they are technological changes. The effects of (iii) are known from the benchmark model. If the matching function changes in such a way to favor one side, it increases the likelihood that trade organized by that side is efficient. If the site cost falls, it increases the likelihood that trade organized by the inelastic side becomes efficient. In the following analysis, we focus on elements (i) and (ii).

With a trading platform, the label of an organizer needs to be clarified. Arguably, the individuals who run the trading platform can be labeled organizers. We do not use the label this way because it is not tied to the difference between the two sides of the market. Instead, we use the label in the same way as in the benchmark model, where the defining features of an organizer are the site cost and the role in the matching function. With a trading platform, since element (i) changes the site cost into a participation cost, the role in the matching function is the only remaining feature that distinguishes an organizer from a visitor. This potential difference between the two sides can be meaningful for a platform. For example, an online job search site may be designed so that one click takes a visitor to a page with several job ads.

Let a be the number of sites per organizer and $\bar{a}$ the upper bound on a. (In the benchmark model, $\bar{a} = 1$.) For an individual of type $i \in \{e, n\}$, let ϕ_i be the fee charged by the platform on an individual if the individual is a visitor. If the individual is an organizer, the fee is $\phi_i + (a-1)\,\psi$, where ψ is the fee per additional site. The participation cost in addition to the platform fee is c_i to an individual of type i.

In market e, the measure of sites is θa and the measure of matches is $F\left(\theta a\right) = M\left(\theta a, 1\right)$. Social welfare is:

$$w_e\left(\theta, a\right) = F\left(\theta a\right) - \left[c_e + \phi_e + (a-1)\,\psi\right]\theta - c_n - \phi_n.$$

The site cost is k to the platform. We assume that the platform is self-financed; that is, $\theta a k = \left[\phi_e + (a-1)\,\psi\right]\theta + \phi_n$. Substituting the constraint, we have:

$$w_e\left(\theta, a\right) = F\left(\theta a\right) - \left(c_e + ak\right)\theta - c_n.$$

Similarly, in market n, the measure of matches is $M\left(a, \theta\right) = \theta F(\frac{a}{\theta})$, and social welfare is:

$$w_n\left(\theta, a\right) = \theta F(\frac{a}{\theta}) - \left[c_n + \phi_n + (a-1)\,\psi\right] - \left(c_e + \phi_e\right)\theta.$$

The self-financing constraint on the platform is $ak = \phi_n + (a-1)\psi + \phi_e \theta$. Substituting this constraint into the welfare function yields:

$$w_n(\theta, a) = \theta F(\frac{a}{\theta}) - c_e \theta - ak - c_n.$$

In market $i \in \{e, n\}$, the planner chooses (θ, a) to maximize $w_i(\theta, a)$ subject to the site capacity constraint $a \le \bar{a}$ and individual rationality constraints.

If $a = 1$, then $w_e(\theta)$ and $w_n(\theta)$ above are the same as their counterparts in the benchmark model (see (2.1) and (2.5)). Therefore, element (i) of the platform has no effect on the efficient allocation. More generally, even if $a \neq 1$, the platform fees (ϕ_e, ϕ_n, ψ) have no effect on the efficient allocation. These fees affect how the site cost is shared by the two sides as part of participation costs. The planner can use transfers between the two sides to neutralize the effect of the platform fees.

To analyze the effect of element (ii), note that the capacity constraint on sites, $a \le \bar{a}$, is binding in the efficient allocation in market e but may not be so in market n. In market e, the measure θ and the number a are both associated with elastic individuals. The two contribute to matches symmetrically in the form θa but have different social marginal costs. For θ, the social marginal cost of inducing entry is the sum of the site cost and the normal participation cost. For a, the marginal cost of increasing sites is only the site cost. If elastic individuals are the organizers, the social marginal benefit of changing θ is equal to the social marginal cost and, hence, exceeds the site cost. This implies that the social marginal benefit of increasing sites for the organizer exceeds the site cost, and so the organizer should use up the capacity. In contrast, in market n, the choices of a and

θ are associated with different sides of the market: a is for inelastic individuals and θ for elastic individuals. The social marginal cost of elastic individuals' entry is not necessarily higher than that of increasing sites for an organizer (inelastic individual). The capacity constraint on sites binds in the efficient allocation if and only if the site cost is not too high, which we assume in the analysis below.[22]

With the binding capacity constraint, the efficient θ is $\frac{1}{\bar{a}}\theta_e(\frac{c_e}{\bar{a}}+k)$ in market e and $\bar{a}\theta_n(c_e)$ in market n, where $\theta_e(.)$ is defined in (2.2) and $\theta_n(.)$ in (2.6). Social welfare in the two markets can be computed, respectively, as

$$W_e = f(\frac{c_e}{\bar{a}}+k) - c_n, \quad W_n = \bar{a}\left[f^{-1}(c_e) - k\right] - c_n.$$

Market e dominates market n if and only if $G(c_e, k) < 0$, where G is redefined as

$$G(c,k) = \bar{a}\left[f^{-1}(c_e) - k\right] - f(\frac{c_e}{\bar{a}}+k).$$

Redefine the threshold $c_d(k,\bar{a})$ as the solution to $f(\frac{c_d}{\bar{a}}+k) = c_d$. Then, $G(c_d, k) = 0$. Under a regularity condition similar to Assumption 3, $G(c_e, k) < 0$ if and only if $c_e > c_d(k, \bar{a})$.

Since f is a decreasing function, $c_d(k,\bar{a})$ is decreasing in k and increasing in $\bar{a}$. An increase in $\bar{a}$ reduces the likelihood of $c_e > c_d(k,\bar{a})$. That is, if the platform increases the capacity of sites per organizer, it increases the relative likelihood that the market organized by the inelastic side becomes efficient.

[22]Precisely, let $x = a/\theta$ and write the capacity constraint as $\bar{a} \geq \theta x$. The Lagrangian of the planner's problem in market n is $[F(x) - c_e - xk]\theta - c_n + \lambda(\bar{a} - \theta x)$, where λ is the Lagragian multiplier of the constraint. The optimal choice x is $x = \theta_n(c_e)$, where $\theta_n(.)$ is the function defined in (2.6). Since $\lambda = F(\theta_n(c_e)) - k$, the capacity constrain is binding if and only if $k < F(\theta_n(c_e))$.

G. Search Effort of Inelastic Individuals

This appendix relaxes the assumption that the supply on the inelastic side is fixed. We endogenize search effort on the inelastic side so that this side is only inelastic relatively to the other side. We show that the results of the baseline model continue to hold with this relative inelasticity.

The measure of inelastic individuals is fixed at one as in the baseline model. However, conditional on participating in the market, each inelastic individual can choose search effort ℓ. The sum of an inelastic individual's cost of participation and search effort is $c_n(\ell)$, where $c_n(0) = 0$, $c_n'(\ell) > 0$, $c_n''(\ell) > 0$ and $c_n'(\infty) = \infty$. Note that the assumption $c_n(0) = 0$ makes non-participation equivalent to the choice $\ell = 0$. Search effort increases the measure of matches. If the inelastic side organizes the market, search effort also increases the measure of trading sites. Precisely, if an inelastic individual is a market organizer, searching with effort ℓ requires the individual to set up ℓ sites. Since the measure of inelastic individuals is one, ℓ is also the total effective search units of inelastic individuals. Redefine θ as the ratio of elastic individuals to ℓ rather than to the measure of inelastic individuals. Then, the measure of elastic individuals is $\theta\ell$. The measure of matches is $M(\theta\ell, \ell)$ in market e and $M(\ell, \theta\ell)$ in market n. As in the baseline model, define $F(\theta) = M(\theta, 1)$. The elastic side is short if $\theta < 1$ and long if $\theta > 1$, although the meaning of θ is modified as above. Whether the matching technology is symmetric, favors the short side, or favors the long side is defined as in section 2.1. The baseline model corresponds to the case where ℓ is fixed at one.

In market e, the sum of the elastic side's costs is $(c_e + k)\,\theta\ell$, the sum of the inelastic side's costs is $c_n(\ell)$, and the sum of output is $M(\theta\ell, \ell)$. Social welfare is

$$w_e(\theta, \ell) = M(\theta\ell, \ell) - (c_e + k)\,\theta\ell - c_n(\ell)\,.$$

In market n, the sum of the elastic side's costs is $c_e\theta\ell$, the sum of the inelastic side's costs is $c_n(\ell) + k\ell$, and the sum of output is $M(\ell, \theta\ell)$. Social welfare is

$$w_n(\theta, \ell) = M(\ell, \theta\ell) - (c_e\theta + k)\,\ell - c_n(\ell)\,.$$

Proposition G.1. *Maintain Assumptions 1 and 3. Replace Assumption 2 by* $c_n'(0) < \max\{f(c_e + k)\,, f^{-1}(c_e) - k\}$ *and* $c_n'(0) < 1 - c_e - k$*. Then, Remark 1, Theorem 3.1 and Theorem 4.1 hold.*

Proof. Consider first the economy with frictional matching. Since the matching function has constant returns to scale, we can rewrite w_e and w_n above as

$$\begin{aligned} w_e(\theta, \ell) &= [F(\theta) - (c_e + k)\,\theta]\,\ell - c_n(\ell) \\ w_n(\theta, \ell) &= \left[\theta F(\tfrac{1}{\theta}) - c_e\theta - k\right]\ell - c_n(\ell)\,. \end{aligned}$$

It is clear that w_e is maximized at $\theta_e(c_e + k)$ given by (2.2), and w_n is maximized at $\theta_n(c_e)$ given by (2.6). These ratios are the same as in the baseline model and are independent of ℓ. Substituting the efficient ratio θ and using the function f defined in (2.4), we express

$$w_e = f(c_e + k)\,\ell - c_n(\ell)\,,\ \ w_n = [f^{-1}(c_e) - k]\ell - c_n(\ell)\,.$$

Efficient search effort is ℓ_e in market e and ℓ_n in market n, which satisfy:

$$c_n'(\ell_e) = f(c_e + k)\,, \quad c_n'(\ell_n) = f^{-1}(c_e) - k.$$

The assumption in the proposition, $c_n'(0) < \max\{f(c_e + k), f^{-1}(c_e) - k\}$, ensures search effort of inelastic individuals to be strictly positive in at least one market. This assumption also ensures at least one market to be viable. If market i is viable, maximized social welfare in the market is

$$W_i = c_n'(\ell_i)\,\ell_i - c_n(\ell_i), \quad i \in \{e, n\}.$$

Since $[c_n'(\ell)\,\ell - c_n(\ell)]$ is increasing in ℓ, then $W_e < W_n$ if and only if $\ell_e < \ell_n$. With the definitions of ℓ_e and ℓ_n above, this condition is equivalent to $f(c_e + k) < f^{-1}(c_e) - k$, i.e., to $G(c_e, k) > 0$ where G is defined in (2.8). Because this condition is the same as in the baseline model, Theorems 3.1 and 4.1 hold.

When matching is frictionless, $M(\theta, 1) = \min\{\theta, 1\}$. In this case, the efficient ratio θ is $\theta = 1$. Social welfare is equal to $\ell - (c_e + k)\,\ell - c_n(\ell)$ in both market e and market n. Under the assumption $c_n'(0) < 1 - c_e - k$ in the proposition, social welfare under frictionless matching is maximized by a unique interior level of search effort. Market e and market n are welfare equivalent in this case, as stated in Remark 1. **QED**